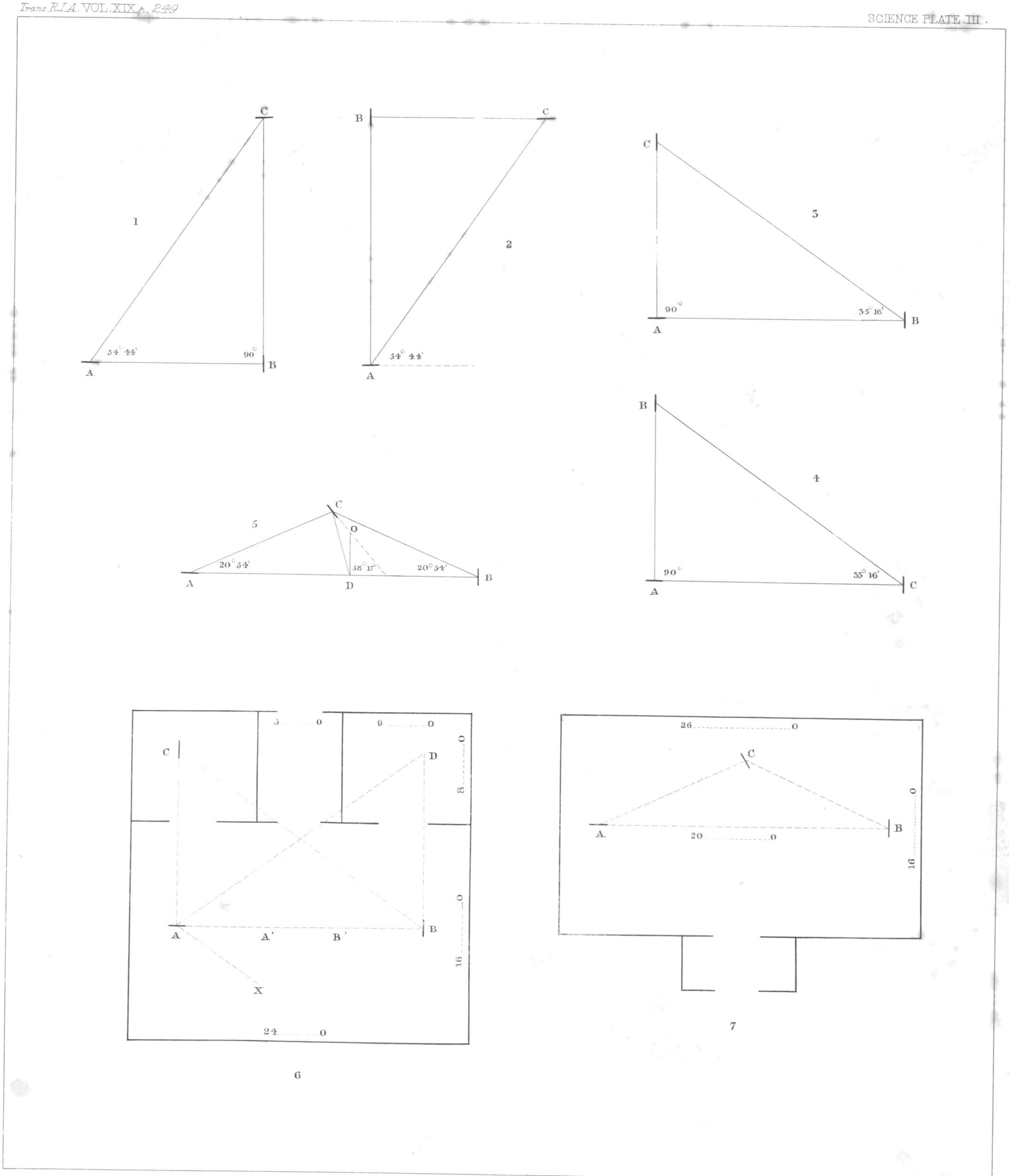

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X. *Supplement to a Paper "On the mutual Action of permanent Magnets, considered chiefly in Reference to their best relative Position in an Observatory." By the Rev.* HUMPHREY LLOYD, *D.D., Fellow of Trinity College, and Professor of Natural Philosophy in the University of Dublin, F.R.S., V.P.R.I.A., Honorary Member of the American Philosophical Society.*

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IN a former paper I have investigated the conditions of equilibrium of the forces exerted upon one another by three magnets, such as those employed in the Dublin Magnetical Observatory, and in the Observatories since established by the British government, in observing the three elements* of the Earth's Magnetic Force. The axes of these magnets being supposed to lie in the same horizontal plane, the forces which they exert upon one another are necessarily directed in that plane; and the conditions of equilibrium of these forces are expressed by *five* equations, the forces exerted upon one of the magnets, in the direction perpendicular to its axis, being destroyed by the reaction of its supports. To fulfil these conditions, there are only *four* arbitrary quantities,—namely, the angles which the lines connecting the centres of the three magnets make with the magnetic meridian, and the azimuth of the axis of one of the magnets. Hence it followed, that complete equilibrium was not attainable, except for determinate values of the relative forces of the magnets. I was, therefore, compelled to select among the conditions of equilibrium, all of which

* These elements are the *declination*, and the *horizontal* and *vertical components* of the force. The magnets employed in observing the first and second of these elements are capable of motion in the *horizontal* plane, the axis of the first being in the magnetic meridian, and that of the second perpendicular to it; the third magnet, being supported on knife-edges, is capable of motion only in a *vertical* plane, and its azimuth is arbitrary.

are not of equal practical value; and I was thus led to consider some less complete solutions of the problem, in which three, or even two only, of these conditions are satisfied.

But all these solutions are exposed to the objection, that the positions of the magnets which fulfil the conditions are dependent upon their relative forces, and are, therefore, subject to vary along with them:—in other words, that upon any change of these forces, the equilibrium already effected will be destroyed, and a new arrangement of the magnets be required to restore it.

To obviate the inconvenience arising from such a displacement of the magnets employed in the observations, it has been suggested to fulfil the conditions of equilibrium by means of additional magnets, whose positions could be readily altered as the forces varied. To this, however, there are serious objections. In the first place, by thus increasing the number of balancing actions, the chances of error in the positions of the centres of force, as well as the liability to alteration in their intensities, are multiplied; and, secondly, on account of this liability to change, no absolute measurement could be safely made, without a re-examination of the relative forces of the magnets, and a readjustment of their positions; so that nothing appears to be gained.

Under all these circumstances, the best course appears to be, to satisfy so many of the conditions of equilibrium, as are capable of being fulfilled *independently of the relative forces of the magnets*, and to apply corrections for the actions which remain unbalanced. In this manner, the changes which the forces of the magnets may undergo, in process of time, will not disturb the equilibrium which has been effected; and the unbalanced actions, being in definite directions, will admit of being determined by an easy experiment, and allowed for by a simple correction.

In order that any one of the equations of equilibrium* may subsist independently of the ratios of the forces of the magnets, the part which contains one of these ratios, and that which is independent of it, must *separately* vanish, and the five equations are resolved into the following:

* Equations (10, 11, 12, 13, 15), pp. 167, 170.

$$3\cos(2\beta-\zeta)+\cos\zeta=0, \qquad \sin 2\gamma=0; \tag{1}$$

$$3\sin(2\beta-\zeta)+\sin\zeta=0, \qquad 1-3\cos 2\gamma=0; \tag{2}$$

$$3\cos(2\alpha-\zeta)+\cos\zeta=0, \qquad 1+3\cos 2\gamma=0; \tag{3}$$

$$3\sin(2\alpha-\zeta)+\sin\zeta=0, \qquad \sin 2\gamma=0; \tag{4}$$

$$3\cos(2\beta-\zeta)+\cos\zeta=0, \qquad 3\sin(2\alpha-\zeta)+\sin\zeta=0. \tag{5}$$

Now it will be seen, on a little consideration, that of these five pairs of equations, the equations (2) and (3) exclude, each, the other four; so that if we fulfil the condition expressed by (2), or that expressed by (3), in this way, we cannot at the same time satisfy *any* other. On the other hand, each pair of the remaining conditions, expressed by the equations (1, 4, 5), has one equation in common; so that for the fulfilment of these three conditions, *three* equations only are to be satisfied; and these three equations are not only not inconsistent, but even leave one of the angles still undetermined.

These equations are

$$\sin 2\gamma=0, \tag{6}$$

$$3\cos(2\beta-\zeta)+\cos\zeta=0, \tag{7}$$

$$3\sin(2\alpha-\zeta)+\sin\zeta=0. \tag{8}$$

The first of them determines the angle γ; and as the other two contain three arbitrary angles, they may be fulfilled in an infinite variety of ways. Accordingly we must have

$$\gamma=0, \text{ or } \gamma=90^\circ; \tag{9}$$

that is, the line connecting the magnets A and B must be *parallel* or *perpendicular to the magnetic meridian.* And the angles, α, β, ζ, which determine the place and azimuth of the third magnet, are connected by the relations,

$$\frac{\frac{1}{3}+\cos 2\beta}{\sin 2\beta}=-\tan\zeta=\frac{\sin 2\alpha}{\frac{1}{3}-\cos 2\alpha}; \tag{10}$$

so that when one of these angles is assumed or given, the other two are determined.

The natural course is to assume the *azimuth* of the magnet C, and thence determine the place of its centre. Let us suppose, then, that the plane of the magnet C is *parallel to the magnetic meridian*, or that

$$\zeta = 0.$$

The equations (7, 8) then give,

$$\cos 2\beta = -\tfrac{1}{3}, \quad \sin 2\alpha = 0\,;$$

and these two equations, together with (6), solve the problem. As we cannot have $\gamma = 0$, $\alpha = 0$, simultaneously, there are two solutions, namely :

$$\left.\begin{array}{ll}\gamma = 0, & \alpha = 90^\circ, \\ \gamma = 90^\circ, & \alpha = 0,\end{array}\right\} \beta = 54^\circ\, 44'.$$

The corresponding arrangements of the magnets are represented in Figs. 1 and 2.

Again, if the plane in which the magnet C is constrained to move be *perpendicular to the magnetic meridian*, or

$$\zeta = 90^\circ,$$

the equations (7, 8) are then reduced to

$$\sin 2\beta = 0, \;\; \cos 2\alpha = \tfrac{1}{3}\,;$$

which, in conjunction with (6), furnish the two solutions:

$$\left.\begin{array}{ll}\gamma = 0, & \beta = 90^\circ, \\ \gamma = 90^\circ, & \beta = 0,\end{array}\right\} \alpha = 35^\circ\, 16'.$$

These arrangements are represented in Figs. 3 and 4.

In estimating the comparative merits of these four arrangements, we should observe that the magnet C is usually much less massive, and therefore less powerful than either of the other two; and, accordingly, that the arrangements represented in Figs. 1 and 4, in which the distance, AB, of the stronger magnets is the shortest side of the triangle ABC, are, on that account, in-

ferior to those represented in Figs. 2 and 3. Of the latter, the arrangement (Fig. 3) is to be preferred, where our object is to diminish as much as possible the residual action upon the declination magnet, A; and, on the other hand, the arrangement (Fig. 2) should be chosen, if we prefer to diminish the action upon the magnet B.

There is still another particular disposition which deserves to be considered: that, namely, in which the magnet C is *equally distant* from the other two. This condition is expressed by the relation,

$$\alpha + \beta = 180^\circ;$$

and eliminating, by means of it, the angle β in (10), we have

$$\frac{\cos 2\alpha + \frac{1}{3}}{\sin 2\alpha} = \frac{\sin 2\alpha}{\cos 2\alpha - \frac{1}{3}};$$

whence $\cos^2 2\alpha - \sin^2 2\alpha = \frac{1}{9}$, $\sin 2\alpha = \pm \frac{2}{3}$, and

$$\alpha = \pm 20^\circ \, 54'.$$

Again, substituting this value in (10), we have

$$\tan \zeta = \frac{\pm 2}{\sqrt{5} - 1} = \pm 1.6180, \qquad \zeta = 58^\circ \, 17', \text{ or } = 180^\circ - 58^\circ \, 17'.$$

Accordingly, the arrangement of the magnets is that represented in Fig. 5, or the reverse arrangement, in which the magnet C is in the corresponding position on the opposite side of the line AB.

Let us now consider, briefly, the corrections required for the residual actions, and the manner in which they are to be experimentally determined.

In virtue of the equations (6) and (7), the action exerted by the magnets B and C upon A, in the magnetic meridian, is null; the disturbing action is, therefore, *perpendicular* to the meridian, and operates only as a *deflecting* force. The amount of the deflection produced by this resultant force is easily determined; for we have only to reverse the magnets B and C simultaneously, and it is obvious that the difference of the readings of the magnet A, in these two positions of the deflecting magnets, is double the deflection sought. In order to

eliminate the actual changes of declination which may occur in the interval of the two parts of the observation, simultaneous observations should be made with an auxiliary apparatus in another apartment; or, should such an apparatus be not at hand, the effect of the changes may be got rid of by making a series of readings of the magnet A, with the deflecting magnets alternately in the two positions. The amount of the deflection, thus determined, is to be applied as a correction in measurements of the *absolute declination*: being a constant quantity, or nearly so, its effect upon the *declination changes* may be disregarded. Lastly, there being no disturbing force upon the magnet A, in the magnetic meridian itself, the *absolute horizontal intensity*, determined by experiments of vibration and deflection, according to the method of Gauss, will need no correction.*

On the other hand, the disturbing force exerted upon the magnet B, by the other two, is directed in the magnetic meridian itself, and therefore *conspires with*, or *opposes*, the force of the earth. The correction required for its action is determined with the same facility as in the former case. We have only to reverse the magnets A and C simultaneously, and to note the change of position of the magnet B thereby produced. Half the change, converted into parts of the whole force by multiplying it by a coefficient already known, is the ratio, $\frac{f}{F}$, of the disturbing force to the total force; and, in order to correct for this force, we have only to multiply the observed results by the coefficient $1 \mp \frac{f}{F}$, using the upper sign when the disturbing action conspires with that of the earth, and the lower when it is opposed to it.

Finally, with respect to the magnet C, the disturbing action, being perpendicular to the plane in which the magnet is constrained to move, is destroyed by the reaction of its supports, and no correction is needed.

* The *resultant* of the force of the earth, and of the disturbing action, will of course differ, theoretically, from the former; but, in general, by an inappreciable amount. If X denote the earth's horizontal force, and δ the deflection produced by the disturbing action, the resultant force will be X secant δ. Now, supposing δ to be *two minutes* (which is greater than any amount it can have with magnets of the size of those employed in the Dublin Observatory, and at the distances recommended below) the resultant force will exceed X by the quantity .0000002X.

It may be useful to suggest, in a few words, the form of building adapted to these arrangements.

For the arrangement represented in Fig. 3, the ground-plan of the building may be a square, whose sides (24 feet in length) are parallel and perpendicular to the magnetic meridian, (Fig. 6). This area may be conveniently divided into four parts, viz.: a principal room, 24 feet in length and 16 feet in width; two subordinate rooms, and a vestibule. The principal room should contain the magnets A and B, which may be placed at an interval of 18 feet,* the joining line being the axis of the room. Two pedestals, A′ and B′, (at an interval of $4\frac{1}{2}$ feet), will serve to support the reading telescopes; and the observer's chair may be placed between them. The magnet C should be placed in one of the small rooms, its distance from the magnet A being $AC = AB \times \text{tang } 35^{\circ} 16' = 18 \times 0.707 = 12.73$ feet. In order to diminish, as far as possible, the deflecting force exerted by the magnets B and C upon A, these magnets should have their poles *similarly* placed (i. e. the same pole in each turned to the east); for, in this case, the resulting action is the *difference* of the forces exerted by the separate magnets.

It will be convenient to fix another pedestal, D, for the support of an inclination instrument, in the second of the small rooms, and at the point corresponding to C in the first;—the line BD being perpendicular to the magnetic meridian, and the distance $BD = AC$. It is manifest that, in this position, the action of the magnets B and C upon a magnetic particle at D will be perpendicular to the magnetic meridian; and will, therefore, have no effect upon the position of the inclination needle, being destroyed by the reaction of its supports. And, in order that the action of the magnet A may be in the same direction, it is only necessary to turn it round, so that its axis may lie in the line AX, which makes with the magnetic meridian an angle $BAX = BAD$. For $\tan D = \sqrt{2}$; and $\tan DAX = \dfrac{2 \tan D}{\tan^2 D - 1} = 2\sqrt{2}$; so that $\tan D = \frac{1}{2} \tan DAX$, and DB is the

* At this distance, the deflection produced by the magnet B upon A, (the deflecting magnet being of the size and power of those employed in the Dublin Magnetical Observatory), is only about $1\frac{1}{2}$ minutes; and the greater part of this small disturbance will be annulled by the opposing action of the magnet C.

direction of the force exerted by the magnet A (in that position) upon the point D. This temporary adjustment of the magnet A may be at once effected by means of a line drawn on the supporting pedestal; and it is obvious that it may be accomplished without removing the magnet from its stirrup, or interfering in any way with its permanent adjustments.

The building required to receive the magnets, in the arrangement represented in Fig. 5, may be still simpler; consisting only of a single room, 26 feet in length, and 16 feet in width, and having a portico with a second door, to prevent draughts of air, (Fig. 7).

To find a suitable place for the inclination instrument, we have only to determine the point on the line AB, at which the action of the magnet C is perpendicular to AB. Then, the action of the magnet B being perpendicular to AB at every point of this line, the forces exerted by B and C will be perpendicular to the meridian, and will therefore be destroyed by the reaction of the supports; and, in order that the same thing should hold also for the magnet A, we have only to turn that magnet, temporarily, into a position perpendicular to the meridian.

Let D (Fig. 5) be the point sought, and DO a line perpendicular to AB; then the condition requires that tan CDO $= \frac{1}{2}$ tan OCD ; or, denoting the angle CDA by x, cotan $x = \frac{1}{2} \tan (x - 58° 17')$. Whence, developing and substituting the value of tan (58° 17′), we have the following quadratic for the determination of tan x,

$$\tan^2 x - 4.854 \tan x - 2 = 0.$$

Accordingly, $\tan x = 5.236$, or $= -0.382$; and $x = 79° 11'$, or $= -20° 54'$. Of these solutions the former is that adapted to the present purpose; the latter giving the point A itself.

The pedestal erected at the point D will likewise serve to support the reading telescope of the magnet B, which may be inserted in a groove cut in the top, so as not to interfere with the other instrument. The supporting pedestal of the telescope of the magnet A should be on the line DA, its centre being four or five feet from the point D, so as to admit the observer's chair between the two pedestals.